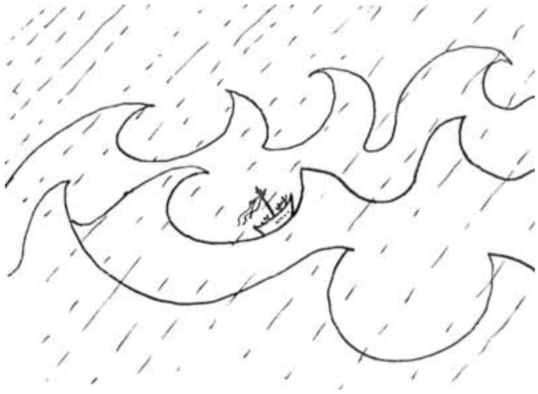
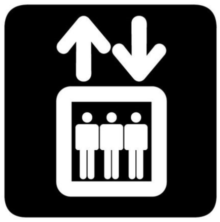
1. Lost at sea

You are lost at sea in the fog, just 10 km from the coast (straight). How you can be sure to find the coast by traveling the least km possible.





2. The lifts

In a tower with 10 floors (0, 1, 2, ... 10), you must develop a 4 lifts management algorithm that minimizes the residents’ latency.
Initially we can assume that people arrive randomly in the day to order a lift.
- It goes down or goes up one floor in 3 seconds.
- Each stop at a floor lasts 15 seconds.

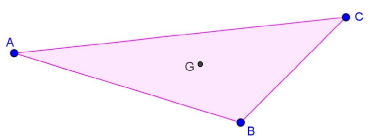
3. Gonfler un triangle

ABC est un triangle, on note *d* la longueur du plus grand côté et G son centre de gravité.
On définit le processus gonflage de la manière suivante :
E_3 est le triangle ABC. On place le point M_4 sur une demi-droite partant de G, le plus loin possible de G et tel que la distance de M_4 à chacun des points de E_3 soit inférieure ou égale à *d*.
E_4 est le polygone convexe contenant E_3 et M_4.
On fait tourner la demi-droite de 1° et on construit le point M_5 sur cette nouvelle demi-droite de la même manière : être le plus loin de G et la distance de M_5 à chaque point de E_4 soit inférieure ou égale à *d*.
E_5 est le polygone convexe contenant E_4 et M_5.
Ainsi de suite.
Que peut-on dire sur E_{363} ?

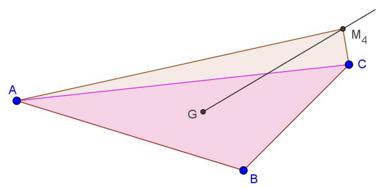
3. Blowing up a triangle

ABC is a triangle, d is its longest side and G its center of gravity. We define the inflating process as follows:
E3 is the triangle ABC. Placing the point M4 on a ray starting from G, as far as possible from G such that the distance between M4 and each of the E3 points is less than or equal to d.
E4 is the convex polygon containing E3 and M4.
The ray is rotated by 1 ° and the point M5 is constructed on this ray in the same way: being the farther from G and distance from M5 to each of E4 points is less than or equal to d.
E5 is the convex polygon containing E4 and M5.
And so on. What can we say about E363?

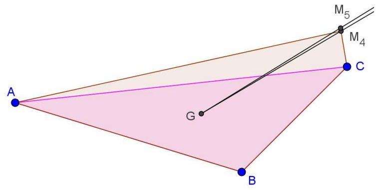
Triangle at the beginning



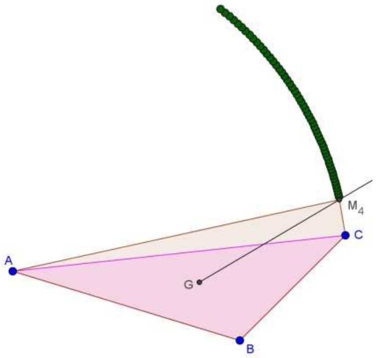
First ray with M4



Ray rotated by 1° and M5



With all the other M points

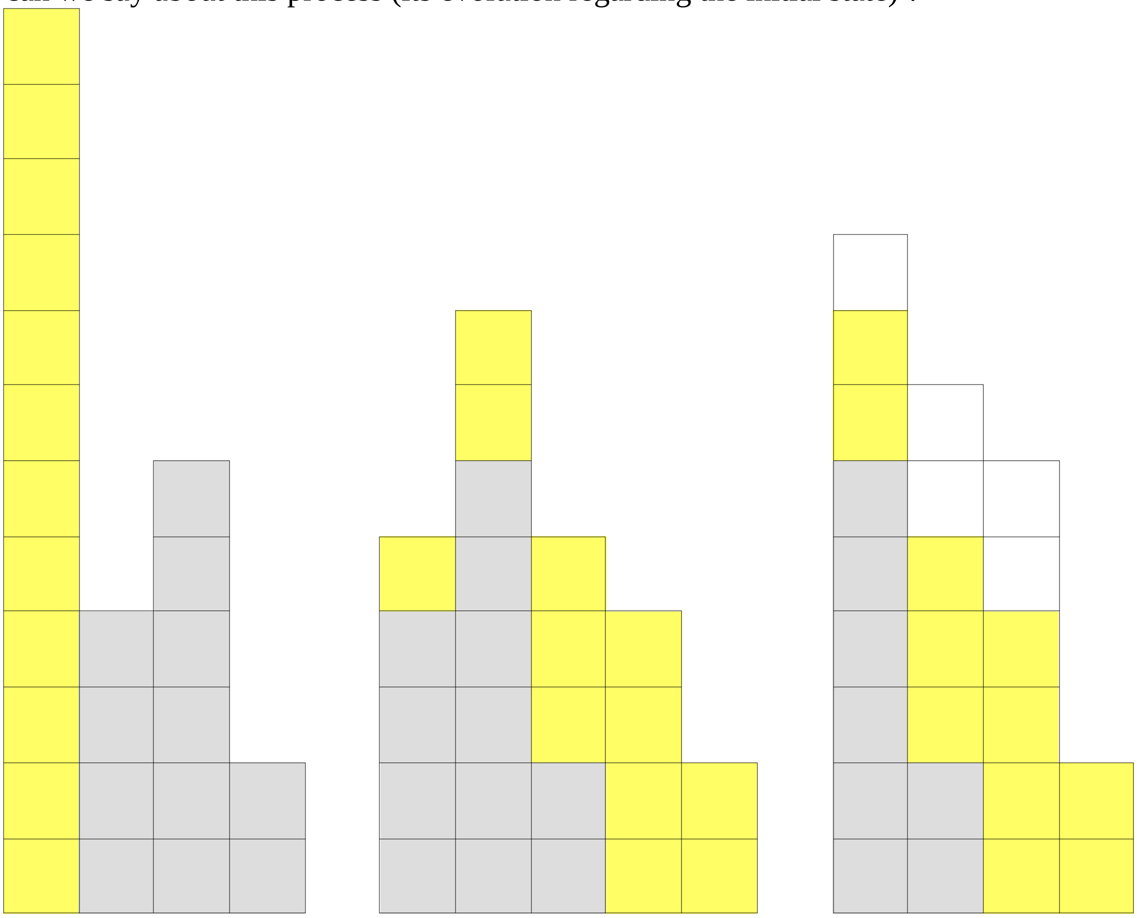


4. Let's move the walls!

A "wall" consists of a finite sequence of "stacks", each with a finite number of "bricks". Consider the transformation of removing the first stack, distributing its bricks on the remaining stacks, one brick on the second stack, two in the third, three in the fourth, etc; within the limit of the number of available blocks. If all existing stacks have been reached before exhausting the bricks contained in the first stack, we continue the process by creating new ones.

Example of an initial state (0) with the first two steps (1 and 2)

What can we say about this process (its evolution regarding the initial state) ?



0 1 2

5. Distance between words

We have a dictionary in which all words have 5 letters from the alphabet {A, B, C, D, E, F, G, H}.
For example: ABCAB or BDEFG

We ask you to define a distance between two words in this dictionary. A distance is an application which associates to any word a positive real number and satisfies:

- For X and Y in the dictionary, $d(X, Y) = d(Y, X)$
- For X and Y in the dictionary, $d(X, Y) = 0 \Leftrightarrow X = Y$
- For X, Y and Z in the dictionary, $d(X,Z) \leqslant d(X,Y)+d(Y,Z)$

6. Filling containers

Three containers A, B and C have respective capacities of 9, 5 and π liters. Initially A contains 9 liters of water, B and C are empty. We are allowed to pour water from a container X into a container Y in the following two conditions:

1) we do not waste water
2) after the pouring, X is empty or Y is full.

Prove that for any $\epsilon>0$, it is possible, after a finite number of pourings, to get into one of the containers 1 liter of water with an ϵ precision.

7. Card trick

We start with a stack of cards ... we only keep (in a 32 or 52 deck) aces, 2, 3, 4, 5, and 6.

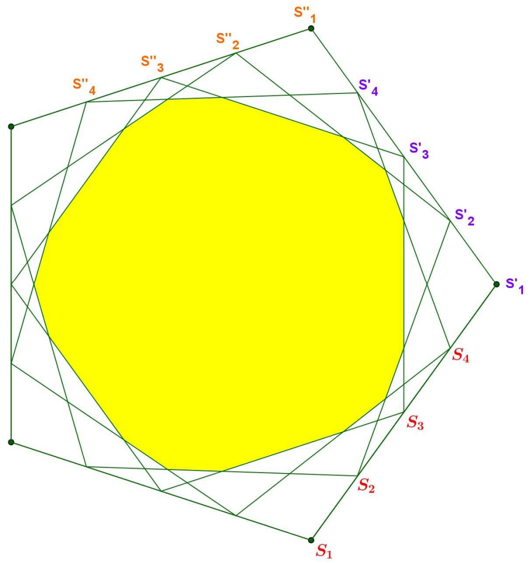
1. We mix this set of 24 cards and arrange them on a table so that they form a circle, one of the cards is distinguished as the first and defines a direction of travel in the circle.
2. The "player" mentally selects one of the first 6 cards, then selects a second card by shifting a number of cards corresponding to the value of the first card and a third by shifting a number of cards corresponding to the value of the second card and so on, until (always mentally), he gets back into the 6 first cards.
3. The "wizard" then guesses the arrival card.

If we assume that magic does not exist, then whatever the first chosen card, the arrival card must be the same! Investigate !

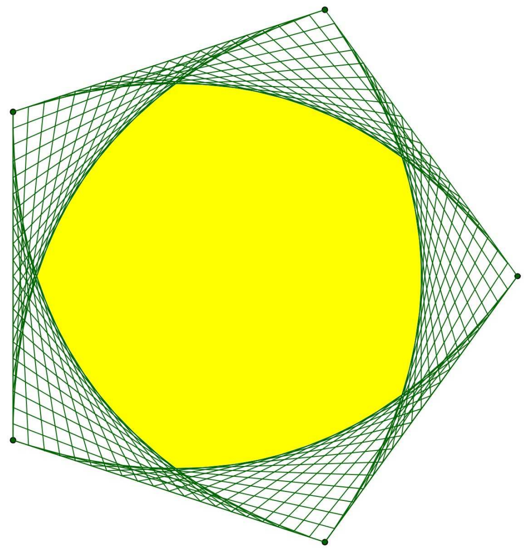
8. Set ? Curves

Each side of a regular polygon is divided into n regular segments.
For each consecutive side, we construct the n segments $[S_1S'_n]$, $[S_2S'_{n-1}]$, $[S_3S'_{n-2}]$...
What can one say of the figure made of these segments?

Regular pentagone with n=4



Regular pentagone with n=20



9. eco-systems

An eco-system is a set of pairs $\{(A_i; \alpha_i), 1 \leqslant i \leqslant n\}$ where A_i is a point of the plane, αi an integer corresponding to the number of round trips between this point and the starting point.
To each starting point M of the plane, we associate the total travel T(M) corresponding to the eco-system : $T(M) = \sum_{i=1}^{n} \alpha_i \times MA_i$
A solution for eco-system is a point S such as the function T is minimum in S.
Solving an eco-system is to find all its solutions.
Try to find results for 2 points and then 3 points eco-systems.

10. Being as far as possible
You are in a rectangular room with your three worst enemies.
Where should you be to be as far away as possible from them three.
The distance from them will be the sum of the distances of each of them to you.