

Comment résoudre une grille de Sudoku avec *Mathematica* ?

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```
In[27]= Import[  
  "http://www.septeuill78.fr/wp-content/uploads/2012/10/sudoku-difficile-3.gif"  
]
```

Out[27]=

	7			1			9	
9			8					7
		3						6
	4				1	5		
	3						1	
		2	7				6	
5						6		
6					5			2
	8			2			7	

```
In[28]= L1 = {Null, 7, Null, Null, 1, Null, Null, 9, Null}
```

```
Out[28]= {Null, 7, Null, Null, 1, Null, Null, 9, Null}
```

```
In[29]= L2 = {9, Null, Null, 8, Null, Null, Null, Null, 7}
```

```
Out[29]= {9, Null, Null, 8, Null, Null, Null, Null, 7}
```

```
In[30]= L3 = {Null, Null, 3, Null, Null, Null, Null, Null, 6}
```

```
Out[30]= {Null, Null, 3, Null, Null, Null, Null, Null, 6}
```

```
In[31]=
```

```
L4 = {Null, 4, Null, Null, Null, 1, 5, Null, Null}
```

```
Out[31]= {Null, 4, Null, Null, Null, 1, 5, Null, Null}
```

```
In[32]= L5 = { Null, 3, Null, Null, Null, Null, Null, 1, Null}
```

```
Out[32]= {Null, 3, Null, Null, Null, Null, Null, 1, Null}
```

```
In[33]= L6 = { Null, Null, 2, 7, Null, Null, Null, 6, Null}
```

```
Out[33]= {Null, Null, 2, 7, Null, Null, Null, 6, Null}
```

```
In[34]= L7 = {5, Null, Null, Null, Null, Null, 6, Null, Null}
```

```
Out[34]= {5, Null, Null, Null, Null, Null, 6, Null, Null}
```

```
In[35]= L8 = {6, Null, Null, Null, Null, Null, 6, Null, Null}
```

```
Out[35]= {6, Null, Null, Null, Null, Null, 6, Null, Null}
```

```
In[36]= L9 = {Null, 8, Null, Null, 2, Null, Null, 7, Null}
```

```
Out[36]= {Null, 8, Null, Null, 2, Null, Null, 7, Null}
```

```
In[37]= MF = MatrixForm
```

```
Out[37]= MatrixForm
```

```
In[38]= (M = {L1, L2, L3, L4, L5, L6, L7, L8, L9}) // MF
```

```
Out[38]/MatrixForm=
```

$$\begin{pmatrix} \text{Null} & 7 & \text{Null} & \text{Null} & 1 & \text{Null} & \text{Null} & 9 & \text{Null} \\ 9 & \text{Null} & \text{Null} & 8 & \text{Null} & \text{Null} & \text{Null} & \text{Null} & 7 \\ \text{Null} & \text{Null} & 3 & \text{Null} & \text{Null} & \text{Null} & \text{Null} & \text{Null} & 6 \\ \text{Null} & 4 & \text{Null} & \text{Null} & \text{Null} & 1 & 5 & \text{Null} & \text{Null} \\ \text{Null} & 3 & \text{Null} & \text{Null} & \text{Null} & \text{Null} & \text{Null} & 1 & \text{Null} \\ \text{Null} & \text{Null} & 2 & 7 & \text{Null} & \text{Null} & \text{Null} & 6 & \text{Null} \\ 5 & \text{Null} & \text{Null} & \text{Null} & \text{Null} & \text{Null} & 6 & \text{Null} & \text{Null} \\ 6 & \text{Null} & \text{Null} & \text{Null} & \text{Null} & \text{Null} & 6 & \text{Null} & \text{Null} \\ \text{Null} & 8 & \text{Null} & \text{Null} & 2 & \text{Null} & \text{Null} & 7 & \text{Null} \end{pmatrix}$$

```
In[39]= tp = Position [M, _Integer]
```

```
Out[39]= {{1, 2}, {1, 5}, {1, 8}, {2, 1}, {2, 4}, {2, 9}, {3, 3},
          {3, 9}, {4, 2}, {4, 6}, {4, 7}, {5, 2}, {5, 8}, {6, 3}, {6, 4},
          {6, 8}, {7, 1}, {7, 7}, {8, 1}, {8, 7}, {9, 2}, {9, 5}, {9, 8}}
```



```
In[43]= (initialise[M]) // MF
```

```
Out[43]/MatrixForm=
```

$$\begin{pmatrix} \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{7\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ \{9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{3\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{4\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{3\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{2\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ \{5\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ \{6\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \\ \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{8\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} & \{1, 2, 3, 4, 5, 6, 7, 8, 9\} \end{pmatrix}$$

```
In[44]=
```

```
filtrePlus[G_] := Map[Function[z,
  Map[If[Length[#] > 1, Complement[#, Apply[Union, Cases[z, {_}]]], #] &, z]], G]
```

```
In[45]= filtreMoins[G_] :=
```

```
Map[Function[l, Fold[If[Length[Position[l, #2]] == 1, ReplacePart[#1,
  First[First[Position[#1, #2]]] -> {#2}], #1] &, l, Range[9]]], G]
```

```
In[46]= filtreMax[pos_] :=
```

```
carré[filtrePlus[carré[Transpose[filtrePlus[Transpose[filtrePlus[pos]]]]]]]
```

```
In[47]= filtreMin[pos_] := carré[
```

```
filtreMoins[carré[Transpose[filtreMoins[Transpose[filtreMoins[pos]]]]]]]
```

```
In[48]= filtre[pos_] := filtreMin[filtreMax[pos]]
```

```
In[49]= P = (filtre[FixedPoint[filtre, initialise[M]]) // MF
```

```
Out[49]/MatrixForm=
```

$$\begin{pmatrix} \{2, 4\} & \{7\} & \{8\} & \{2, 3, 4, 5, 6\} & \{1\} & \{2, 3, 4, 5, 6\} & \{ \\ \{9\} & \{6\} & \{1, 4, 5\} & \{8\} & \{3, 4, 5\} & \{2, 3, 4, 5\} & \{1 \\ \{2, 4\} & \{1, 2, 5\} & \{3\} & \{2, 4, 5, 9\} & \{4, 5, 7, 9\} & \{2, 4, 5, 7, 9\} & \{1 \\ \{7\} & \{4\} & \{6, 9\} & \{2, 3, 6, 9\} & \{3, 6, 8, 9\} & \{1\} & \\ \{8\} & \{3\} & \{5, 6, 9\} & \{2, 4, 5, 6, 9\} & \{4, 5, 6, 9\} & \{2, 4, 5, 6, 9\} & \\ \{1\} & \{5, 9\} & \{2\} & \{7\} & \{3, 4, 5, 8, 9\} & \{3, 4, 5, 8, 9\} & \{3 \\ \{5\} & \{1, 2, 9\} & \{1, 4, 7, 9\} & \{1, 3, 4, 9\} & \{3, 4, 7, 8, 9\} & \{3, 4, 7, 8, 9\} & \\ \{6\} & \{1, 2, 9\} & \{1, 4, 7, 9\} & \{1, 3, 4, 5, 9\} & \{3, 4, 5, 7, 8, 9\} & \{3, 4, 5, 7, 8, 9\} & \\ \{3\} & \{8\} & \{1, 4, 9\} & \{1, 4, 5, 6, 9\} & \{2\} & \{4, 5, 6, 9\} & \{ \end{pmatrix}$$

```

In[50]= TrouveHyp[P_] := With[{Q = {Map[Function[1, Length[Flatten[1, 1]]], P],
  Map[Function[1, Length[Flatten[1, 1]]], Transpose[P]]}},
  With[{m = Min[Select[Flatten[Q], # > 9 &]}],
    With[
      {p = First[{If[Position[Q, m][[1, 1]] == 2, {RandomChoice[Position[Transpose[
        P][[Position[Q, m][[1, 2]]]], Min[Select[Map[Length, Transpose[
          P][[Position[Q, m][[1, 2]]]], # > 1 &]]], RandomChoice[
            Transpose[P][[Position[Q, m][[1, 2]]]][[RandomChoice[Position[
              Transpose[P][[Position[Q, m][[1, 2]]]], Min[Select[Map[Length,
                Transpose[P][[Position[Q, m][[1, 2]]]], # > 1 &]]]]]]],
          {RandomChoice[RandomChoice[Transpose[P][[Position[Q, m][[
            1, 2]]]][[RandomChoice[Position[Transpose[P][[
              Position[Q, m][[1, 2]]]], Min[Select[Map[Length, Transpose[
                P][[Position[Q, m][[1, 2]]]], # > 1 &]]]]]]]}},
          {RandomChoice[Position[P][[Position[Q, m][[1, 2]]]],
            Min[Select[Map[Length, P][[Position[Q, m][[1, 2]]]], # > 1 &]]],
          RandomChoice[P][[Position[Q, m][[1, 2]]]][[RandomChoice[
            Position[P][[Position[Q, m][[1, 2]]]], Min[Select[Map[Length,
              P][[Position[Q, m][[1, 2]]]], # > 1 &]]]]], {RandomChoice[
            RandomChoice[P][[Position[Q, m][[1, 2]]]][[RandomChoice[
              Position[P][[Position[Q, m][[1, 2]]]], Min[Select[Map[Length,
                P][[Position[Q, m][[1, 2]]]], # > 1 &]]]]]]]}]}][[1]]],
      Hypo[Extract[P, p], p, First[If[Position[Q, m][[1, 1]] == 2,
        {RandomChoice[
          Position[Transpose[P][[Position[Q, m][[1, 2]]]], Min[Select[
            Map[Length, Transpose[P][[Position[Q, m][[1, 2]]]], # > 1 &]]],
          RandomChoice[Transpose[P][[Position[Q, m][[1, 2]]]][[
            RandomChoice[Position[Transpose[P][[Position[Q, m][[1, 2]]]],
              Min[Select[Map[Length, Transpose[P][[Position[Q, m][[1, 2]]]],
                # > 1 &]]]]]]], {RandomChoice[RandomChoice[
                Transpose[P][[Position[Q, m][[1, 2]]]][[RandomChoice[Position[
                  Transpose[P][[Position[Q, m][[1, 2]]]], Min[Select[Map[Length,
                    Transpose[P][[Position[Q, m][[1, 2]]]], # > 1 &]]]]]]]}},
          {RandomChoice[Position[P][[Position[Q, m][[1, 2]]]],
            Min[Select[Map[Length, P][[Position[Q, m][[1, 2]]]], # > 1 &]]],
          RandomChoice[P][[Position[Q, m][[1, 2]]]][[
            RandomChoice[Position[P][[Position[Q, m][[1, 2]]]], Min[
              Select[Map[Length, P][[Position[Q, m][[1, 2]]]], # > 1 &]]]]],
          {RandomChoice[RandomChoice[P][[Position[Q, m][[1, 2]]]][[RandomChoice[
            Position[P][[Position[Q, m][[1, 2]]]], Min[Select[Map[Length,
              P][[Position[Q, m][[1, 2]]]], # > 1 &]]]]]]]}]}][[3]]]]]]]
    ]
  ]
]

```

```

In[51]= Applique[hyp_, pos_] := ReplacePart[pos, hyp[[2]] → {hyp[[3]]}]

```

```
In[52]= (FixedPoint[filtre, initialise[M]]) // MF
```

```
Out[52]/MatrixForm=
```

{2, 4}	{7}	{8}	{2, 3, 4, 5, 6}	{1}	{2, 3, 4, 5, 6}	{
{9}	{6}	{1, 4, 5}	{8}	{3, 4, 5}	{2, 3, 4, 5}	{1
{2, 4}	{1, 2, 5}	{3}	{2, 4, 5, 9}	{4, 5, 7, 9}	{2, 4, 5, 7, 9}	{1
{7}	{4}	{6, 9}	{2, 3, 6, 9}	{3, 6, 8, 9}	{1}	
{8}	{3}	{5, 6, 9}	{2, 4, 5, 6, 9}	{4, 5, 6, 9}	{2, 4, 5, 6, 9}	
{1}	{5, 9}	{2}	{7}	{3, 4, 5, 8, 9}	{3, 4, 5, 8, 9}	{3
{5}	{1, 2, 9}	{1, 4, 7, 9}	{1, 3, 4, 9}	{3, 4, 7, 8, 9}	{3, 4, 7, 8, 9}	
{6}	{1, 2, 9}	{1, 4, 7, 9}	{1, 3, 4, 5, 9}	{3, 4, 5, 7, 8, 9}	{3, 4, 5, 7, 8, 9}	
{3}	{8}	{1, 4, 9}	{1, 4, 5, 6, 9}	{2}	{4, 5, 6, 9}	{

```
In[53]= TrouveHyp[FixedPoint[filtre, initialise[M]]]
```

```
Out[53]= Hypo[{2, 4}, {3, 1}, 2]
```

```
In[54]= Applique[TrouveHyp[FixedPoint[filtre, initialise[M]]],  
FixedPoint[filtre, initialise[M]]] // MF
```

```
Out[54]/MatrixForm=
```

{4}	{7}	{8}	{2, 3, 4, 5, 6}	{1}	{2, 3, 4, 5, 6}	{
{9}	{6}	{1, 4, 5}	{8}	{3, 4, 5}	{2, 3, 4, 5}	{1
{2, 4}	{1, 2, 5}	{3}	{2, 4, 5, 9}	{4, 5, 7, 9}	{2, 4, 5, 7, 9}	{1
{7}	{4}	{6, 9}	{2, 3, 6, 9}	{3, 6, 8, 9}	{1}	
{8}	{3}	{5, 6, 9}	{2, 4, 5, 6, 9}	{4, 5, 6, 9}	{2, 4, 5, 6, 9}	
{1}	{5, 9}	{2}	{7}	{3, 4, 5, 8, 9}	{3, 4, 5, 8, 9}	{3
{5}	{1, 2, 9}	{1, 4, 7, 9}	{1, 3, 4, 9}	{3, 4, 7, 8, 9}	{3, 4, 7, 8, 9}	
{6}	{1, 2, 9}	{1, 4, 7, 9}	{1, 3, 4, 5, 9}	{3, 4, 5, 7, 8, 9}	{3, 4, 5, 7, 8, 9}	
{3}	{8}	{1, 4, 9}	{1, 4, 5, 6, 9}	{2}	{4, 5, 6, 9}	{

```
In[55]= (S = FixedPoint[filtre, Applique[TrouveHyp[FixedPoint[filtre, initialise[M]]],  
FixedPoint[filtre, initialise[M]]]]) // MF
```

```
Out[55]/MatrixForm=
```

{2}	{7}	{8}	{3, 4, 5, 6}	{1}	{3, 4, 5, 6}	{3
{9}	{6}	{1, 5}	{8}	{3, 4, 5}	{2, 3, 4, 5}	{1, 2
{4}	{1, 5}	{3}	{2, 5, 9}	{5, 7, 9}	{2, 5, 7, 9}	{1,
{7}	{4}	{6, 9}	{2, 3, 6, 9}	{3, 6, 8, 9}	{1}	{
{8}	{3}	{5, 6, 9}	{2, 4, 5, 6, 9}	{4, 5, 6, 9}	{2, 4, 5, 6, 9}	{
{1}	{5, 9}	{2}	{7}	{3, 4, 5, 8, 9}	{3, 4, 5, 8, 9}	{3, 4
{5}	{1, 2, 9}	{1, 4, 7, 9}	{1, 3, 4, 9}	{3, 4, 7, 8, 9}	{3, 4, 7, 8, 9}	{
{6}	{1, 2, 9}	{1, 4, 7, 9}	{1, 3, 4, 5, 9}	{3, 4, 5, 7, 8, 9}	{3, 4, 5, 7, 8, 9}	{
{3}	{8}	{1, 4, 9}	{1, 4, 5, 6, 9}	{2}	{4, 5, 6, 9}	{1,

```
In[56]=
```

On cherche à savoir si l'hypothèse est correcte, c'est-à-dire il ne reste qu'une seule solution. S'il y a contradiction, c'est-à-dire, le nombre de solutions = 0, alors nous revenons sur la décision de l'hypothèse. Il faut donc mettre en place quelque chose de la sorte:

```
In[57]= etape[hyp_, pos_] := etape[nouvellehyp[hyp], nouvellepos[pos]]
```

Fonction permettant de connaître le nombre de solutions restantes :

```
In[58]= Apply[Times, Map[Apply[Times, Map[Length, S[#]]]] &, Range[9]]
```

```
Out[58]= 10 353 518 498 122 039 296 000 000 000
```

En prenant en compte que lorsque l'on trouve 0 avec cette fonction il y a contradiction (un ensemble vide), nous devons donc dans ce cas revenir sur l'hypothèse précédente